1. A crime is committed by one of two suspects, A and B. Initially, there is equal evidence against both of them. In further investigation at the crime scene, it is found that the guilty party had a blood type found in 10% of the population. Suspect A does match this blood type, whereas the blood type of Suspect B is unknown. (a) Given this new information, what is the probability that A is the guilty party? (b) Given this new information, what is the probability that B’s blood type matches that found at the crime scene?

Solution

To solve this problem, we will use Bayesian reasoning.

### Part (a): Probability that A is the Guilty Party

Let:

- \( G\_A \) be the event that A is guilty.

- \( G\_B \) be the event that B is guilty.

- \( E \) be the event that the blood type found at the crime scene matches that of the guilty party.

We are asked to find the probability that A is guilty given the evidence \( E \), i.e., \( P(G\_A | E) \).

Initially, there is equal evidence against both suspects, so the prior probabilities are:

\[

P(G\_A) = P(G\_B) = \frac{1}{2}

\]

Given the blood evidence:

- If A is guilty, then the evidence \( E \) will definitely match A's blood type, so \( P(E | G\_A) = 1 \).

- If B is guilty, there is only a 10% chance that the blood type found matches B's blood type, so \( P(E | G\_B) = 0.1 \).

Using Bayes' theorem:

\[

P(G\_A | E) = \frac{P(E | G\_A) \cdot P(G\_A)}{P(E)}

\]

Where \( P(E) \) is the total probability of the evidence:

\[

P(E) = P(E | G\_A) \cdot P(G\_A) + P(E | G\_B) \cdot P(G\_B)

\]

Substituting the values:

\[

P(E) = (1) \cdot \left(\frac{1}{2}\right) + (0.1) \cdot \left(\frac{1}{2}\right) = \frac{1}{2} + \frac{0.1}{2} = \frac{1.1}{2} = 0.55

\]

Now, substituting \( P(E) \) back into Bayes' theorem:

\[

P(G\_A | E) = \frac{1 \cdot \frac{1}{2}}{0.55} = \frac{0.5}{0.55} = \frac{10}{11} \approx 0.909

\]

So, the probability that A is the guilty party given the evidence is approximately \*\*0.909 (90.9%)\*\*.

### Part (b): Probability that B’s Blood Type Matches That Found at the Crime Scene

Now, we are asked to find the probability